Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2017

Mathematics

Higher level

Paper 2

18 pages

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Instructions to Examiners

Abbreviations

M   Marks awarded for attempting to use a valid **Method**; working must be seen.

(M)   Marks awarded for **Method**; may be implied by **correct** subsequent working.

A   Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding *M* marks.

(A)   Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R   Marks awarded for clear **Reasoning**.

N   Marks awarded for **correct** answers if **no** working shown.

AG   Answer given in the question and so no marks are awarded.

Using the markscheme

1   General
Mark according to RM™ Assessor instructions and the document **"Mathematics HL: Guidance for e-marking May 2017"**. It is essential that you read this document before you start marking. In particular, please note the following.

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp *A0* by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by RM™ Assessor.

2   Method and Answer/Accuracy marks
- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award *M0* followed by *A1*, as *A* mark(s) depend on the preceding *M* mark(s), if any.
- Where *M* and *A* marks are noted on the same line, for example, *M1A1*, this usually means *M1* for an **attempt** to use an appropriate method (for example, substitution into a formula) and *A1* for using the **correct** values.
- Where the markscheme specifies *(M2)*, *N3*, *etc*, do **not** split the marks.

- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final **A1**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct **FT** working shown, award **FT** marks as appropriate but do not award the final **A1** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final **A1** *(ignore the further working)*
2.	$\dfrac{1}{4}\sin 4x$	$\sin x$	Do not award the final **A1**
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final **A1**

3 **N** marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of **N** and other marks.
- There may be fewer **N** marks available than the total of **M**, **A** and **R** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets**, for example, **(M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6   Misread

*If a candidate incorrectly copies information from the question, this is a misread (**MR**).*
*A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate*
that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an
***M** mark, but award all others so that the candidate only loses **[1 mark]**.*

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7   Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does
*not cover the work seen. In such cases the annotation DM should be used and a brief **note** written*
next to the mark explaining this decision.

8   Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question
specifies a method, other correct methods should be marked in line with the markscheme. If in
doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9   Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

   Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives:

   $$f'(x) = \left(2\cos(5x - 3)\right)5 \quad \left(= 10\cos(5x - 3)\right) \qquad\qquad \boldsymbol{A1}$$

   Award **A1** for $\left(2\cos(5x - 3)\right)5$, even if $10\cos(5x - 3)$ is not seen.

10   Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11   Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12   Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13   More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

14.   Candidate work

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. This work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

> **Note:** Accept all numerical answers which round correctly to the correct 2 sf answer unless stated otherwise. Do not accept any answer given to 2 sf unless stated otherwise.

Section A

1.   (a) $P(5 \text{ or more}) = \dfrac{29}{75}(= 0.387)$ *(M1)A1*

[2 marks]

   (b)   mean score $= \dfrac{2 \times 3 + 3 \times 15 + 4 \times 28 + 5 \times 17 + 6 \times 9 + 7 \times 3}{75}$ *(M1)*

   $= \dfrac{323}{75}(= 4.31)$ *A1*

[2 marks]

Total [4 marks]

2.   (a)   **METHOD 1**

   $4x^2 + y^2 = 7$

   $8x + 2y\dfrac{dy}{dx} = 0$ *(M1)(A1)*

   $\dfrac{dy}{dx} = -\dfrac{4x}{y}$

> **Note:** Award **M1A1** for finding $\dfrac{dy}{dx} = -2.309\ldots$ using any alternative method.

   hence gradient of normal $= \dfrac{y}{4x}$ *(M1)*

   hence gradient of normal at $\left(1, \sqrt{3}\right)$ is $\dfrac{\sqrt{3}}{4}(= 0.433)$ *(A1)*

   hence equation of normal is $y - \sqrt{3} = \dfrac{\sqrt{3}}{4}(x - 1)$ *(M1)A1*

   $\left(y = \dfrac{\sqrt{3}}{4}x + \dfrac{3\sqrt{3}}{4}\right)$ $\left(y = 0.433x + 1.30\right)$

continued…

Question 2 continued

METHOD 2

$$4x^2 + y^2 = 7$$

$$y = \sqrt{7 - 4x^2}$$ **(M1)**

$$\frac{dy}{dx} = -\frac{4x}{\sqrt{7 - 4x^2}}$$ **(A1)**

Note: Award **M1A1** for finding $\frac{dy}{dx} = -2.309\ldots$ using any alternative method.

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hence gradient of normal $= \dfrac{\sqrt{7 - 4x^2}}{4x}$ **(M1)**

hence gradient of normal at $\left(1, \sqrt{3}\right)$ is $\dfrac{\sqrt{3}}{4} (= 0.433)$ **(A1)**

hence equation of normal is $y - \sqrt{3} = \dfrac{\sqrt{3}}{4}(x - 1)$ **(M1)A1**

$$\left(y = \frac{\sqrt{3}}{4}x + \frac{3\sqrt{3}}{4} \right) \left(y = 0.433x + 1.30 \right)$$

[6 marks]

(b)   Use of $V = \pi \displaystyle\int_0^{\frac{\sqrt{7}}{2}} y^2 \, dx$

$$V = \pi \int_0^{\frac{\sqrt{7}}{2}} \left(7 - 4x^2 \right) dx$$ **(M1)(A1)**

Note: Condone absence of limits or incorrect limits for **M** mark.
Do not condone absence of or multiples of π.

$$= 19.4 \left(= \frac{7\sqrt{7}\,\pi}{3} \right)$$ **A1**

[3 marks]

Total [9 marks]

3.   (a) $P(X < 250) = 0.0228$ **(M1)A1**

                                                 [2 marks]

   (b) $\dfrac{250 - \mu}{1.5} = -2.878\ldots$ **(M1)(A1)**

         $\Rightarrow \mu = 254.32$ *A1*

   > **Notes:** Only award **A1** here if the correct 2dp answer is seen.
   >   Award **M0** for use of 1.5^2.

                                                 [3 marks]

   (c) $\dfrac{250 - 253}{\sigma} = -2.878\ldots$ **(A1)**

         $\Rightarrow \sigma = 1.04$ *A1*

                                                 [2 marks]

                                          Total [7 marks]

4.   (a) $k^2 - k - 12 < 0$

              $(k - 4)(k + 3) < 0$ **(M1)**

              $-3 < k < 4$ *A1*

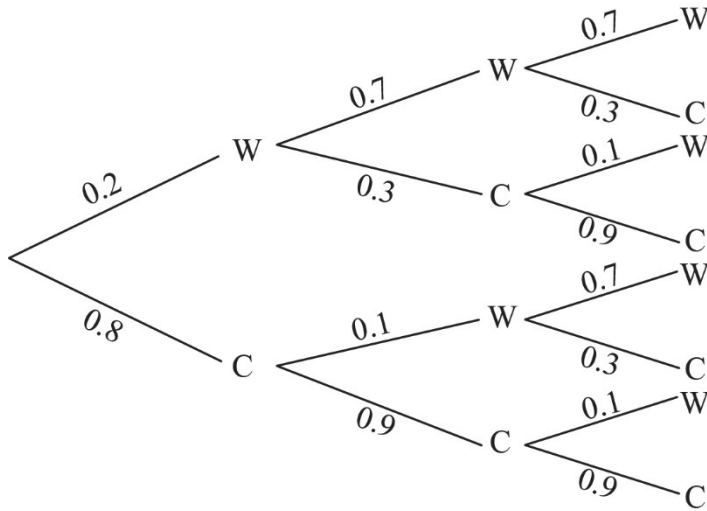   *[2 marks]*

   (b) $\cos B = \dfrac{2^2 + c^2 - 4^2}{4c}$ $\left(\text{or } 16 = 2^2 + c^2 - 4c\cos B\right)$ *M1*

         $\Rightarrow \dfrac{c^2 - 12}{4c} < \dfrac{1}{4}$ *A1*

         $\Rightarrow c^2 - c - 12 < 0$
         from result in (a)
         $0 < AB < 4$ or $-3 < AB < 4$ **(A1)**
         but AB must be at least 2
         $\Rightarrow 2 < AB < 4$ *A1*

   > **Note:**   Allow $\leq AB$ for either of the final two **A** marks.

                                                 [4 marks]

                                          Total [6 marks]

5.   (a)

M1A2

Note: Award *M1* for 3 stage tree-diagram, *A2* for 0.8,0.9,0.3 probabilities correctly placed.

[3 marks]

(b) $0.2 \times 0.7 \times 0.3 + 0.2 \times 0.3 \times 0.9 + 0.8 \times 0.1 \times 0.3 + 0.8 \times 0.9 \times 0.9 = 0.768$

(M1)A1

[2 marks]

(c)   P(1st July is calm | 3rd July is windy) $= \dfrac{\text{P(1st July is calm and 3rd July is windy)}}{\text{P(3rd July is windy)}}$

(M1)

$= \dfrac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{1 - 0.768}$

OR $\dfrac{0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}{0.2 \times 0.7 \times 0.7 + 0.2 \times 0.3 \times 0.1 + 0.8 \times 0.1 \times 0.7 + 0.8 \times 0.9 \times 0.1}$

OR $\dfrac{0.128}{0.232}$

(A1)(A1)

Note: Award *A1* for correct numerator, *A1* for correct denominator.

$= 0.552$

A1

[4 marks]

Total [9 marks]

6. $\log_{10} \dfrac{1}{2\sqrt{2}}(p + 2q) = \dfrac{1}{2}\left(\log_{10} p + \log_{10} q\right)$

   $\log_{10} \dfrac{1}{2\sqrt{2}}(p + 2q) = \dfrac{1}{2}\log_{10} pq$ **(M1)**

   $\log_{10} \dfrac{1}{2\sqrt{2}}(p + 2q) = \log_{10}(pq)^{\frac{1}{2}}$ **(M1)**

   $\dfrac{1}{2\sqrt{2}}(p + 2q) = (pq)^{\frac{1}{2}}$ **(A1)**

   $(p + 2q)^2 = 8pq$

   $p^2 + 4pq + 4q^2 = 8pq$

   $p^2 - 4pq + 4q^2 = 0$

   $(p - 2q)^2 = 0$ **M1**

   hence $p = 2q$ **A1**

                                                                                                                         [5 marks]

7.   **METHOD 1**

$a \times b = b \times c$

$(a \times b) - (b \times c) = 0$

$(a \times b) + (c \times b) = 0$ ***M1A1***

$(a + c) \times b = 0$ ***A1***

$(a + c)$ is parallel to $b \Rightarrow a + c = sb$ ***R1AG***

> **Note:** Condone absence of arrows, underlining, or other otherwise "correct" vector notation throughout this question.

> **Note:** Allow "is in the same direction to", for the final ***R*** mark.

METHOD 2

$$a \times b = b \times c \Rightarrow \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} = \begin{pmatrix} b_2 c_3 - b_3 c_2 \\ b_3 c_1 - b_1 c_3 \\ b_1 c_2 - b_2 c_1 \end{pmatrix}$$ ***M1A1***

$a_2 b_3 - a_3 b_2 = b_2 c_3 - b_3 c_2 \Rightarrow b_3 (a_2 + c_2) = b_2 (a_3 + c_3)$

$a_3 b_1 - a_1 b_3 = b_3 c_1 - b_1 c_3 \Rightarrow b_1 (a_3 + c_3) = b_3 (a_1 + c_1)$

$a_1 b_2 - a_2 b_1 = b_1 c_2 - b_2 c_1 \Rightarrow b_2 (a_1 + c_1) = b_1 (a_2 + c_2)$

$$\frac{(a_1 + c_1)}{b_1} = \frac{(a_2 + c_2)}{b_2} = \frac{(a_3 + c_3)}{b_3} = s$$ ***A1***

$\Rightarrow a_1 + c_1 = sb_1$

$\Rightarrow a_2 + c_2 = sb_2$

$\Rightarrow a_3 + c_3 = sb_3$

$$\Rightarrow \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = s \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$ ***A1***

$\Rightarrow a + c = sb$ ***AG***

[4 marks]

8. **METHOD 1**

consideration of all papers

all papers may be sat in $18!$ ways   *A1*

number of ways of positioning "pairs" of science subjects $= 3! \times 17!$ *A1*

but this includes two copies of each "triple"   *(R1)*

number of ways of positioning "triplets" of science subjects $= 3! \times 16!$ *A1*

hence number of arrangements is $18! - 3! \times 17! + 3! \times 16!$ *M1A1*

$$\left(= 4.39 \times 10^{15}\right)$$

METHOD 2

consideration of all the non-science papers   *(M1)*

hence all non-science papers can be sat in $15!$ ways   *A1*

there are $16 \times 15 \times 14 \ (= 3360)$ ways of positioning the three science papers   *(M1)A1*

hence the number of arrangements is $16 \times 15 \times 14 \times 15! \left(= 4.39 \times 10^{15}\right)$ *(M1)A1*

METHOD 3

consideration of all papers

all papers may be sat in $18!$ ways   *A1*

number of ways of positioning exactly two science subjects $= 3! \times 15! \times 16 \times 15$ *M1A1*

number of ways of positioning "triplets" of science subjects $= 3! \times 16!$ *A1*

hence number of arrangements is $18! - 3! \times 16! - 3! \times 15! \times 16 \times 15$ *M1A1*

$$\left(= 4.39 \times 10^{15}\right)$$

[6 marks]

Section B

9. (a) $\overrightarrow{BC} = (i + 3j + 3k) - (2i - j + 2k) = -i + 4j + k$ *(A1)*

$r = (2i - j + 2k) + \lambda(-i + 4j + k)$

(or $r = (i + 3j + 3k) + \lambda(-i + 4j + k)$)   *(M1)A1*

> **Note:** Do not award *A1* unless $r =$ or equivalent correct notation seen.

[3 marks]

(b) attempt to write in parametric form using two different
parameters **AND** equate   *M1*

$2\mu = 2 - \lambda$

$\mu = -1 + 4\lambda$

$-2\mu = 2 + \lambda$ *A1*

attempt to solve first pair of simultaneous equations for two parameters   *M1*

solving first two equations gives $\lambda = \dfrac{4}{9}$, $\mu = \dfrac{7}{9}$ *(A1)*

substitution of these two values in third equation   *(M1)*

since the values do not fit, the lines do not intersect   *R1*

> **Note:** Candidates may note that adding the first and third equations immediately leads to a
> contradiction and hence they can immediately deduce that the lines do not intersect.

[6 marks]

(c) **METHOD 1**

plane is of the form $r \cdot (2i + j - 2k) = d$ *(A1)*

$d = (i + 3j + 3k) \cdot (2i + j - 2k) = -1$ *(M1)*

hence Cartesian form of plane is $2x + y - 2z = -1$ *A1*

METHOD 2

plane is of the form $2x + y - 2z = d$ *(A1)*

substituting $(1, 3, 3)$ (to find gives $2 + 3 - 6 = -1$)   *(M1)*

hence Cartesian form of plane is $2x + y - 2z = -1$ *A1*

[3 marks]

continued…

Question 9 continued

(d)   **METHOD 1**

attempt scalar product of direction vector BC with normal to plane   *M1*
$(-\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}) \cdot (2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}) = -2 + 4 - 2$
$= 0$ *A1*
hence BC lies in Π_1 *AG*

METHOD 2

substitute eqn of line into plane   *M1*

line $r = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$. Plane $\pi_1 : 2x + y - 2z = -1$

$2(2 - \lambda) + (-1 + 4\lambda) - 2(2 + \lambda)$
$= -1$ *A1*
hence BC lies in Π_1 *AG*

> **Note:** Candidates may also just substitute $2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ into the plane since they are told C lies on π_1.

> **Note:** Do not award *A1FT*.

[2 marks]

(e)   **METHOD 1**

applying scalar product to $\overrightarrow{OA}$ and $\overrightarrow{OB}$ *M1*
$(2\boldsymbol{j} + \boldsymbol{k}) \cdot (2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}) = 0$ *A1*
$(2\boldsymbol{j} + \boldsymbol{k}) \cdot (2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}) = 0$ *A1*

METHOD 2

attempt to find cross product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ *M1*

plane Π_2 has normal $\overrightarrow{OA} \times \overrightarrow{OB} = -8\boldsymbol{j} - 4\boldsymbol{k}$ *A1*

since $-8\boldsymbol{j} - 4\boldsymbol{k} = -4(2\boldsymbol{j} + \boldsymbol{k})$, $2\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to the plane Π_2 *R1*

[3 marks]

(f)   plane Π_3 has normal $\overrightarrow{OA} \times \overrightarrow{OC} = 9\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k}$ *A1*

[1 mark]

continued…

Question 9 continued

(g)   attempt to use dot product of normal vectors   **(M1)**

$$\cos\theta = \frac{(2\boldsymbol{j} + \boldsymbol{k})\bullet(9\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k})}{\left|2\boldsymbol{j} + \boldsymbol{k}\right|\left|9\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k}\right|}$$ **(M1)**

$$= \frac{-11}{\sqrt{5}\sqrt{170}}\left(= -0.377...\right)$$ **(A1)**

> **Note:** Accept $\dfrac{11}{\sqrt{5}\sqrt{170}}$.

acute angle between planes $= 67.8°\left(= 1.18^{c}\right)$ ***A1***

[4 marks]

Total [22 marks]

10. (a) $\displaystyle\int_{0}^{4}\left(\frac{x^{2}}{a} + b\right)\mathrm{d}x = 1 \Rightarrow \left[\frac{x^{3}}{3a} + bx\right]_{0}^{4} = 1 \Rightarrow \frac{64}{3a} + 4b = 1$ ***M1A1***

$$\int_{2}^{4}\left(\frac{x^{2}}{a} + b\right)\mathrm{d}x = 0.75 \Rightarrow \frac{56}{3a} + 2b = 0.75$$ ***M1A1***

> **Note:** $\displaystyle\int_{0}^{2}\left(\frac{x^{2}}{a} + b\right)\mathrm{d}x = 0.25 \Rightarrow \frac{8}{3a} + 2b = 0.25$ could be seen/used in place of either of
> the above equations.

evidence of an attempt to solve simultaneously
(or check given a, b values are consistent)   ***M1***

$a = 32$, $b = \dfrac{1}{12}$ ***AG***

[5 marks]

(b) $\mathrm{E}(X) = \displaystyle\int_{0}^{4} x\left(\frac{x^{2}}{32} + \frac{1}{12}\right)\mathrm{d}x$ **(M1)**

$$\mathrm{E}(X) = \frac{8}{3}\left(= 2.67\right)$$ ***A1***

[2 marks]

(c) $\mathrm{E}\left(X^{2}\right) = \displaystyle\int_{0}^{4} x^{2}\left(\frac{x^{2}}{32} + \frac{1}{12}\right)\mathrm{d}x$ **(M1)**

$$\mathrm{Var(X)} = \mathrm{E}\left(X^{2}\right) - \left[\mathrm{E}(X)\right]^{2} = \frac{16}{15}\left(= 1.07\right)$$ ***A1***

[2 marks]

continued…

Question 10 continued

(d) $\displaystyle\int_0^m \left(\frac{x^2}{32} + \frac{1}{12} \right) dx = 0.5$ **(M1)**

$\dfrac{m^3}{96} + \dfrac{m}{12} = 0.5 \left(\Rightarrow m^3 + 8m - 48 = 0 \right)$ **(A1)**

$m = 2.91$ **A1**

[3 marks]

(e) $Y \sim B(8, 0.75)$ **(M1)**

$E(Y) = 8 \times 0.75 = 6$ **A1**

[2 marks]

(f) $P(Y \geq 3) = 0.996$ **A1**

[1 mark]

Total [15 marks]

11. (a) $g(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$

$g(1) = 0 \Rightarrow a + b = 8$ **M1A1**

$g(-1) = 0 \Rightarrow -a + b = -6$ **A1**

$\Rightarrow a = 7,\ b = 1$ **A1**

[4 marks]

(b) $3x^4 + 7x^3 + x^2 - 7x - 4 = \left(x^2 - 1 \right)\left(px^2 + qx + r \right)$

attempt to equate coefficients   **(M1)**

$p = 3,\ q = 7,\ r = 4$ **(A1)**

$3x^4 + 7x^3 + x^2 - 7x - 4 = \left(x^2 - 1 \right)\left(3x^2 + 7x + 4 \right)$

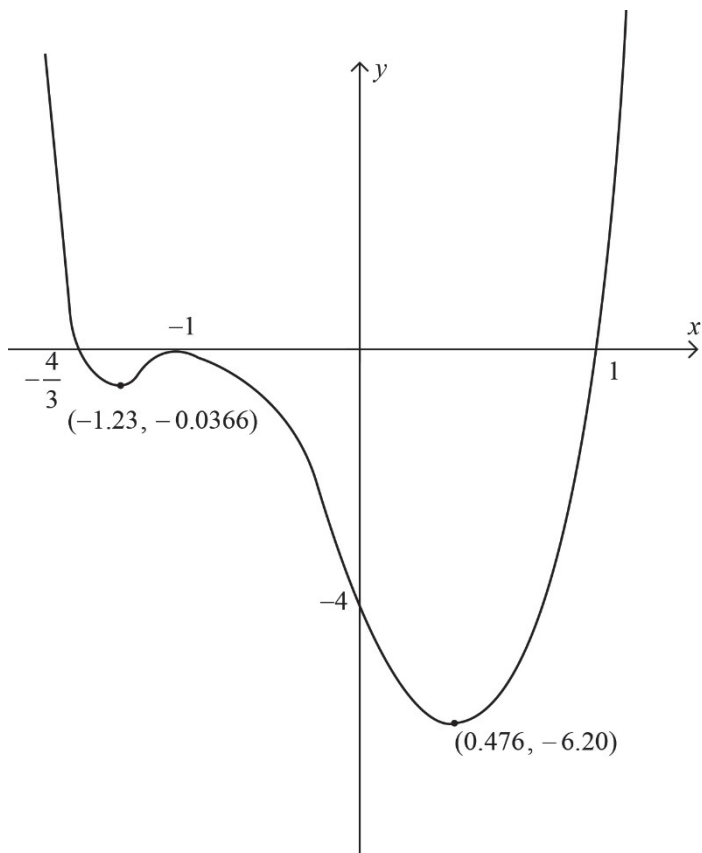$= (x - 1)(x + 1)^2 (3x + 4)$ **A1**

Note: Accept any equivalent valid method.

[3 marks]

continued…

Question 11 continued

(c)

A1 for correct shape (*ie* with correct number of max/min points)
A1 for correct x and y intercepts
A1 for correct maximum and minimum points

[3 marks]

(d) $c > 0$ *A1*
        $-6.20 < c < -0.0366$ *A1A1*

Note:	Award *A1* for correct end points and *A1* for correct inequalities.

Note:	If the candidate has misdrawn the graph and omitted the first minimum point, the maximum mark that may be awarded is *A1FTA0A0* for $c > -6.20$ seen.

[3 marks]

Total [13 marks]